MATH 245 S21, Exam 2 Questions

(60 minutes, open book, open notes)

1. (Question 1 is just instructions; this is a weird requirement of Gradescope)
2. Prove that $\forall x \in \mathbb{R}, \exists! y \in \mathbb{R},\ (x = \lfloor x \rfloor + y) \wedge (0 \leq y < 1)$.
3. Use the division algorithm to prove that $\forall n \in \mathbb{N},\ \frac{n^2+9n+20}{2} \in \mathbb{Z}$.
4. Use (some form of) mathematical induction to prove that $\forall n \in \mathbb{N},\ \frac{n^2+9n+20}{2} \in \mathbb{Z}$.
5. Solve the recurrence given by $a_0 = 2,\ a_1 = 3,\ a_n = -4a_{n-1} - 4a_{n-2}$ $(n \geq 2)$.
6. Let $a_n = n^{1.9} + n^{2.1}$. Prove or disprove that $a_n = O(n^2)$.
7. Let F_n denote the Fibonacci numbers. Prove that $\forall n \in \mathbb{N}_0,\ F_{2n+1}^2 - F_{2n+2}F_{2n} = 1$.